

The new  $(g - 2)_\mu$  measurement and limits on the light Higgs bosons in 2HDM (II)

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(Dated: February 1, 2008)

We discuss how new data for  $a_\mu \equiv (g - 2)_\mu/2$  improve constraints on new physics. Using two types of estimations of  $a_\mu^{had}$  (Davier & Höcker (case A) and Jegerlehner2000 (case B)) we evaluate 95% CL intervals for a new contribution, which can be used to constrain any model beyond the Standard Model. We apply these intervals to the general 2HDM ("Model II"), where up to now one light neutral Higgs particle,  $h$  or  $A$ , was allowed by the data. Assuming that only one Higgs boson contributes to  $a_\mu^{2HDM}$  the two-loop calculation based on the case A leads to the exclusion of scalar  $h$  while a pseudoscalar with mass between 10 and 70 GeV (for  $\tan \beta \geq 20$ ) is allowed. For case B the upper limits for  $\tan \beta$  for both scalar and pseudoscalar are obtained.

PACS numbers: PACS number(s): 12.60.Fr, 14.80.Cp

## I. INTRODUCTION

The precision measurement of  $g - 2$  for the muon is expected to shed light on "new physics". The new E821 result based on 1999 data [1] leads to a current mean of experimental results for  $(g - 2)_\mu$  (from [1])

$$a_\mu^{exp} \equiv \frac{(g - 2)_\mu^{exp}}{2} = 11\,659\,203\,(15) \cdot 10^{-10},$$

where the accuracy of this result (in parentheses) approaches the size of electro-weak contribution,  $a_\mu^{EW}$ . The ultimate accuracy of the E821 experiment is  $4 \cdot 10^{-10}$ .

The QED and EW contributions to  $a_\mu^{SM}$  are well under control. The predictions for the hadronic contribution  $a_\mu^{had}$ , which is more than forty times larger than  $a_\mu^{EW}$ , differ considerably among themselves both for the central value and its uncertainty. This uncertainty is presently of order  $(7 - 10) \cdot 10^{-10}$ , with the dominant error coming from the  $\alpha^2$  vacuum polarization contribution. Useful discussion of various estimations of  $a_\mu^{had}$  can be found in [2], see also more recent papers [3, 4, 5, 6, 7, 8, 9].

The difference between the experimental data,  $a_\mu^{exp}$ , and the Standard Model (SM) prediction [2-47],  $a_\mu^{SM}$ , defines the room for "new physics". Obviously the uncertainties of the hadronic contributions influence the estimation of a size of new effects. To illustrate the present situation we calculate 95% CL intervals ( $lim(95\%)$ ) for an allowed new contribution,  $\delta a_\mu$ , using two representative SM predictions [2]: one based on calculation of  $a_\mu^{had}$  with the  $\alpha^2$  vacuum polarization estimation by Davier and Höcker [29, 30] and the other by Jegerlehner [26, 27]. We derive also intervals which may be relevant for models leading to a positive contribution only ( $lim_+(95\%)$ ).

The obtained intervals we apply to constrain the parameters of the CP conserving 2HDM ("Model II") [48, 49]. This model is based on the two doublets of

complex scalar fields, and predicts existence of five Higgs particles: two neutral Higgs scalars  $h$  and  $H$ , one neutral pseudoscalar  $A$ , and a pair of charged Higgses  $H^\pm$ . In MSSM, which has a Model II Higgs sector, the mass limits (95%CL) from LEP experiments are, *e.g.* for a maximal  $M_h$  scan:  $M_h(M_A)$  greater than 90.0 (91.9) GeV and  $\tan \beta < 0.5$  or  $\tan \beta > 2.4$  [50]. The present 95 % CL limit on the SM Higgs mass is:  $M_h > 114.1$  GeV [50]. The SM Higgs particle with mass around 115 GeV (corresponding to a maximum likelihood ratio from LEP data [50]) contributes to the  $a_\mu^{SM}$  at the level of  $10^{-11}$  (two-loop result [45]).

In the non-supersymmetric 2HDM (II), which we study here, one light neutral Higgs boson  $h$  or  $A$  with mass below 50 GeV is still allowed by data [50-77], see *e.g.* the LEP results from Higgs-strahlung and  $hA$  pair production [51, 52], from the Yukawa processes  $Z \rightarrow f\bar{f}h(A)$  with  $f = \tau$  or  $b$  [54, 55] and the process  $Z \rightarrow h(A)\gamma$  [56], see also [57, 58]. The dedicated fit to the EW precision LEP data performed within this model, even with masses of  $h$  or  $A$  below 20 GeV, is equally good as the corresponding fit in the SM or MSSM [59]. Unfortunately a potential of the HERA collider, discussed in [60, 61], has never been explored to put limits on very light Higgs bosons of the 2HDM (II). Future Linear Collider experiments are not expected either to close a light Higgs window in the 2HDM (II) [62], even these planned to run with a very high luminosity, once more at the Z-peak (GigaZ) [63]. On the other hand one should keep in mind that some theoretical arguments disfavor the Higgs scalar  $h$  with mass below 90 GeV in 2HDM(II) [64].

A light Higgs 'could conceivable evade discovery at LEP and yet show up in a analysis of a low energy data' as  $(g - 2)_\mu$ , as it was pointed out in paper [65]. We have used previous experimental data and the SM prediction(s) to constrain 2HDM (II) in [66]. That one-loop analysis led to a very small improvement in comparison

to LEP limits. It was expected that with increasing precision the  $(g - 2)_\mu$  measurements would lead to more stringent constraints. As we have pointed out in [66], the sign of the one-loop 2HDM (II) contributions to  $a_\mu$  is correlated with the type of the lightest particle:  $h$ ,  $A$  or  $H^\pm$ . We have described the condition which would lead to the exclusion at 95 % CL of a light  $h$  or  $A$ . This condition is presently fulfilled for a pseudoscalar if the DH result for  $a_\mu^{had}$  is used in the one-loop approach, see also [78, 79]. However, as it was pointed out in [80, 81], the two-loop calculation leads to very different results.

In this study we constrain 2HDM (II) by the new data [1] using the two-loop calculation. We apply two different hadronic contributions  $a_\mu^{had}$  as mentioned above (DH=case A and J2000=case B) and two types of 95% CL intervals ( $lim$  and  $lim_+$ ). Obtained constraints we combine with constraints from other processes.

The one-loop analysis of the 2HDM(II) with results which partly overlap with results of this paper can be found in [78], see also [79]. The two-loop analysis, which we follow here, is given in [80, 81]. Related study can be found in [82]. Studies of  $g - 2$  for muon within the context of supersymmetric models were performed earlier [83, 84], many new analyses have appeared recently [85, 86, 87, 88]. Dark matter problem in supersymmetric models is discussed in [89]. Relations to neutrino masses and mixing are studied within supersymmetric and non-supersymmetric models in [90]. Other new analyses made recently are [91], analyses based on the general 2HDM (Model III) are presented in [82, 92]. Analyses of some aspects of cosmic rays can be found in [93]. Relations between the electric and magnetic dipole moments are studied in [94]. An model-independent analysis was done in [95].

In Sec.II we discuss the new  $(g - 2)_\mu$  data and derive limits for a new contribution which can be used in any model beyond the SM. In Sec. III we apply the obtained limits to 2HDM (II) and derive constraints on the parameters of the model. Next we combine these constraints with other experimental information. We study separately the one-loop and two-loop results. Sec. IV contains conclusions.

## II. NEW G-2 DATA FOR THE MUON

### A. New $(g - 2)_\mu$ results

The current world average experimental data on  $(g - 2)$  for muon averaged over the sign of the muon electric charge is given by ( from [2]):

$$a_\mu^{exp} = 116\,592\,023 \cdot 10^{-11}, \quad \text{with } \sigma_{exp} = 151 \cdot 10^{-11}.$$

The Standard Model prediction for this quantity consists of the QED, hadronic and EW contributions:

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{EW}.$$

The QED results can be found in [18, 19]. Hadronic contributions were obtained in: the leading vacuum polarization term (v.p.1) [5, 6, 8, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] and [33, 34], the higher order vacuum polarization term (v.p.2) [20, 22, 28, 35], the light-on-light term (lbl) [5, 8, 9, 25, 36, 37, 38, 39]. The EW results are given in [12, 13, 40, 41, 42, 43, 44, 45, 46, 47]. A useful compilation of recent results is given in [2], see also [10, 11].

The error of the hadronic contribution dominates the total error of the SM predictions. Moreover the hadronic contributions calculated by various authors as discussed in [2, 3, 4, 5, 6, 7, 8, 9] differ strongly leading to distinct estimations of a size of the SM contribution and therefore also of the 'beyond SM' effect. All of these estimations can be divided into two classes, depending on whether the SM prediction for  $(g - 2)_\mu$  is or is not in agreement with the data. We think that it is sensible to study separately consequences of these two classes of the SM predictions. It allows to illustrate the present sensitivity to  $a_\mu^{had}$ , (see [66] for previous results based on a similar approach,) moreover it may serve as a guide for the future results, which obviously will belong to one of these two classes.

In the analysis we consider two representative cases: case A based on Davier and Höcker calculation of the leading vacuum polarization diagram (v.p.1) [29, 30] and case B based on the corresponding Jegerlehner calculation [26, 27], with a smaller and larger hadronic contribution (and its uncertainty), respectively. In the DH analysis the  $e^+e^-$  and  $\tau$ -decay data are used, while J2000 uses only the  $e^+e^-$  data. The higher order hadronic (v.p.2) contribution and the light-on-light contribution are taken from [22, 35] and [37, 38], respectively. In the table below we collect, following [2], the corresponding SM contributions (and their uncertainties).

case	A [in $10^{-11}$ ]		B [in $10^{-11}$ ]	
QED	116 584 706	(3)	116 584 706	(3)
had	6 739	(67)	6 803	(114)
EW	152	(4)	152	(4)
tot	116 591 597	(67)	116 591 661	(114)

The difference between two predictions of the SM, case A and case B, is  $64 \cdot 10^{-11}$ , with the corresponding difference in accuracies equal to  $47 \cdot 10^{-11}$ . It is worthwhile to compare these numbers to the ultimate accuracy of the E821 experiment  $40 \cdot 10^{-11}$ .

Results based on other estimations of the hadronic contribution, can be easily obtained from results for considered cases, see discussion below.

### B. The room for new physics

Here we present results of calculation of the difference between the experimental and theoretical SM results for

$(g-2)_\mu$ , which can be used for any model going beyond the SM. First we derive  $\Delta a_\mu$ , equal to the difference of the central values,  $a_\mu^{exp} - a_\mu^{SM} \equiv \Delta a_\mu$ , and the error  $\sigma$  for this quantity. Knowing  $\Delta a_\mu$  and  $\sigma$  one can calculate an allowed, at chosen confidence level (CL), interval of an additional contribution. For the purpose of this study it is enough to obtain  $\sigma$  by adding in quadrature the corresponding experimental and theoretical errors ( $\sigma_{exp}$  and  $\sigma_{tot}$ ) and to assume a Gaussian distribution. Under these assumptions we calculate in both cases, A and B, the  $\delta a_\mu$  regions, symmetric around  $\Delta a_\mu$ , allowed at 95% CL.

case	A [in $10^{-11}$ ]	B [in $10^{-11}$ ]
$\Delta a_\mu(\sigma)$	426(165)	362(189)
$lim(95\%)$	$102 \leq \delta a_\mu \leq 750$	$-8.65 \leq \delta a_\mu \leq 733$
positive $\delta a_\mu$	99.5%	97.2%
$lim_+(95\%)$	$109 \leq \delta a_\mu \leq 744$	$28.5 \leq \delta a_\mu \leq 696$

In the standard approach one obtains intervals called  $lim(95\%)$ , see the above table for results. We see that although at the one sigma level, *i.e.* for the interval  $\Delta a_\mu \pm \sigma$ , the allowed additional contribution to  $a_\mu$  in cases A and B are of a positive sign only, at the  $2\sigma$  level or 95 %CL the more conservative estimation of the hadronic contribution to  $a_\mu^{SM}$  (case B) makes the negative  $\delta a_\mu$  possible. The SM prediction lies within the 95% CL interval for case B, while for case A it is outside the corresponding interval.

This difference leads to different forms of the limits for a new contribution. The 95 %CL interval leads in case A to an *allowed positive* contribution (*an allowed band*) and at the same time to the *exclusion* of the negative contribution (at the higher CL level, see below). For the case B, the positive (negative) contribution is only bounded from above (below) (*upper limits* for the absolute value of the new contribution). That means that presently the accuracy of the theoretical predictions for the hadronic contribution (both  $a_\mu^{had}$  and  $\sigma_{had}$  matter !) influences in qualitative way the constraints on the new physics [99].

Now we discuss consequences of the present  $(g-2)_\mu$  data for models which can give contribution of only *one* sign. We see (the table above) that for both A and B cases a negative  $\delta a_\mu$  contribution is very unlikely: a positive (negative) contribution corresponds to 99.5 (0.5) % CL for A, while for case B to 97.2 (2.8) % CL. At this level models leading to *only negative*  $\delta a_\mu$  can be *excluded* or saying differently, within models which give a definite sign contribution to  $a_\mu$  only these which give a *positive*  $\delta a_\mu$  can be *realized* at 95% (or higher) CL.

To obtain the allowed range of parameters of such a model we calculate the corresponding 95% CL intervals normalized to the positive contributions only [97]. Results for these intervals, called  $lim_+(95\%)$ , are presented in the last row of the table above [100]. Of course, these two types of 95 % CL intervals,  $lim$  or  $lim_+$ , will lead to very similar constraints of parameters of the model for case A, since they will be obtained from only slightly

different, due to the 0.5% change in normalization, allowed  $\delta a_\mu$ . However, significantly different constraints will arise for case B. Instead of upper limits obtained in the  $lim(95\%)$  approach, in the  $lim_+(95\%)$  method an *allowed band* for a positive contribution is obtained.

We observe that the maximal positive  $\delta a_\mu$  values differ less than 10 % for all discussed cases (696 to 750 in  $10^{-11}$ ). The minimal positive  $\delta a_\mu$  obtained using  $lim_+(95\%)$  method differ by a factor 3 in cases A and B, 109 and 28.5 in  $10^{-11}$ , respectively.

Other recent estimations of the hadronic contributions lead to the following  $\Delta a_\mu(\sigma)$  and intervals  $\delta a_\mu$  in [ $10^{-11}$ ]:

Reference		$\Delta a_\mu(\sigma)$	$lim(95\%)$
Jegerlehner[6]	J2001	376(186)	$12 \leq \delta a_\mu \leq 740$
Narison	[5] N	375(170)	$41.8 \leq \delta a_\mu \leq 708$
Melnikov	[7] M	377(216)	$-47.2 \leq \delta a_\mu \leq 801$
DeTroconiz			
–Yndurain[8]	TY1	363(184)	$2.52 \leq \delta a_\mu \leq 723$
	TY2	338(171)	$3.08 \leq \delta a_\mu \leq 673$
Prades	[9] P	403(169)	$71.8 \leq \delta a_\mu \leq 734$

Note similarities in the obtained  $\Delta a_\mu$  values and at the same time large differences in estimated uncertainties  $\sigma$  for the first three analyses [101]. The first two analyses are similar to case A, while the third one has properties of the case B. The maximal  $\delta a_\mu$  differ from the corresponding numbers for cases A and B within 10% , the minimal  $\delta a_\mu$  differ much more.

The results of the newer analyses also are presented in the above table. The TY1 analysis which corresponds to a (v.p.1) calculation based on the (new)  $e^+e^-$  data, and TY2 where both the  $e^+e^-$  and  $\tau$  decay data are included [8], the P results are based on the weighted average (for v.p.1) of the averaged estimation of (J2001 and TY1) and (DH and TY2) [9]. Note that all these estimations of  $a_\mu^{had}$  are based on a Chiral Model for the light-by-light contribution [37, 38]. The Quark Model gives very different results. The corresponding limits were derived in [8]: the estimations TY3 and TY4 (analogous to TY1 and TY2, *i.e.* without and with  $\tau$  decay data) leading to  $-177 \leq \delta a_\mu \leq 547$  and  $-177 \leq \delta a_\mu \leq 497$  in [ $10^{-11}$ ], respectively.

In the following analysis we will apply intervals obtained for case A and B, a simple rescaling allows to translate the final constraints to results relevant for any other present or future estimations of  $\delta a_\mu$ .

### III. CONSTRAINING THE 2HDM (II)

#### A. A model

In the non-supersymmetric CP conserving 2HDM the Higgs sector contains the two neutral scalars,  $h$  and  $H$ , pseudoscalar  $A$  and charged Higgs bosons  $H^\pm$ . Beside

their masses, three parameters:  $\tan\beta$ , which is the ratio of the vacuum expectation values of the Higgs doublets  $v_2/v_1$ , the mixing angle in the neutral Higgs sector  $\alpha$ , and in addition one more parameter, *e.g.* the Higgs self-coupling  $g_{hH^+H^-}$ , specify the model.

In the Model (II) implementation of the 2HDM, one doublet of fundamental scalar fields couples to the  $u$ -type quarks, the other to the  $d$ -type quarks and charged leptons (this way FCNC processes are avoided at the tree level) [48]. The ratios, relative to the SM values, of the direct coupling constants of the Higgs boson  $h$  or  $H$  to the massive gauge bosons  $V = W$  or  $Z$ , and to fermions (*i.e.* Yukawa couplings) can be determined via angles  $\alpha$  and  $\beta$  [48, 49]. For  $\chi_i^h \equiv g_i^h/(g_i^h)_{SM}$  (and similarly for  $H$ ) we have, in form suitable for discussion simultaneously of  $h$  and  $H$ :

$$\chi_V^h = \sin(\beta - \alpha), \quad \chi_V^H = \cos(\beta - \alpha), \quad (1)$$

$$\chi_u^h = \chi_V^h + \cot\beta\chi_V^H, \quad \chi_u^H = \chi_V^H - \cot\beta\chi_V^h, \quad (2)$$

$$\chi_d^h = \chi_V^h - \tan\beta\chi_V^H, \quad \chi_d^H = \chi_V^H + \tan\beta\chi_V^h, \quad (3)$$

with  $(\chi_V^h)^2 + (\chi_V^H)^2 = 1$  [49]. A very useful *pattern relation* among these couplings holds for both  $h$  and  $H$  [49]:

$$(\chi_u + \chi_d)\chi_V = 1 + \chi_u\chi_d. \quad (4)$$

For  $\chi_V^h = 1$  all couplings of  $h$  have the SM values, couplings of  $H$  to gauge bosons are equal to zero while couplings of  $H$  to fermions may differ considerably from the SM values, for small or large  $\tan\beta$  [102], [103]. For  $\chi_V^H = 1$  the  $H$  is SM-like while  $h$  has different properties (1-3), *e.g.*  $\chi_d^h$  can be very large for large  $\tan\beta$ . From (2) and (3) or (4) it follows that for  $\chi_V = 0$  one obtains  $\chi_u\chi_d = -1$ .

For the pseudoscalar there is no coupling to  $W/Z$ . The Yukawa couplings to fermions  $\chi_d^A(\chi_u^A)$  contain  $\tan\beta$  ( $\cot\beta$ ) factor. The  $\chi_d^A$  is large (small) for large (small)  $\tan\beta$  value, respectively, with the opposite pattern for  $\chi_u^A$ .

In this analysis the Yukawa coupling  $\chi_d$ , relevant for a Higgs boson coupling to a muon, plays a basic role. It is equal to  $\tan\beta$  for a pseudoscalar and  $H^+$  and, if in addition  $\chi_V = \sin(\beta - \alpha) = 0$ , also for a scalar (more precisely  $\chi_d^h = \pm \tan\beta$ ).

At the two-loop level there appears a possibility of having a charged Higgs boson in a loop (see below). A coupling of  $H^+$  to a scalar  $h$  has a form:

$$\chi_{H^+}^h = (1 - \frac{M_h^2}{2M_{H^+}^2})\chi_V^h + \frac{M_h^2 - \mu^2}{2M_{H^+}^2}(\chi_d^h + \chi_u^h), \quad (5)$$

with the normalization as for an elementary charged scalar particle in the SM. For  $\chi_V^h = 0$  one gets

$$\chi_{H^+}^h = \frac{M_h^2 - \mu^2}{2M_{H^+}^2}(\chi_d^h + \chi_u^h) = \frac{M_h^2 - \mu^2}{2M_{H^+}^2}\chi_d^h(1 - (\frac{1}{\chi_d^h})^2). \quad (6)$$

## B. Existing constraints

Main constraints of the parameters of the 2HDM (II) come from LEP experiments, see also a discussion in [67]. From the  $Z \rightarrow Zh$  process upper limits on  $\sin^2(\beta - \alpha)$  were derived [51, 52, 53]. From the tightest limits [53] it follows that  $\sin^2(\beta - \alpha)$  should be smaller than 0.1 for the  $0 \lesssim M_h \lesssim 50$  GeV, and even below 0.01 for a lighter scalar. The data for the cross section for the pair  $(h, A)$  production, proportional to  $\cos^2(\beta - \alpha)$ , when combined with the  $Z \rightarrow Zh$  data leads to an exclusion of a small mass region in the  $(M_h, M_A)$  plane [51, 52]. According to these data the 2HDM (II) may accommodate a very light ( $\lesssim 45$  GeV) neutral scalar  $h$ , with small  $\sin(\beta - \alpha)$ , or a very light pseudoscalar  $A$  as long as:  $M_h + M_A \gtrsim 50$  [51, 52], see also the newest results [58].

A neutral Higgs particle at LEP I has also been searched for in the Yukawa process,  $e^+e^- \rightarrow f\bar{f}h(A)$ , where  $f$  means here  $b$  quark or  $\tau$  lepton. For a light scalar this is an additional, and if  $\beta = \alpha$ , the most important source of information. A (still preliminary) ALEPH analysis of the Yukawa process for a pseudoscalar [54] led to the exclusion at 95% CL for the  $(\tan\beta, M_A)$  plane, allowing for a large  $\tan\beta$ , above 20, for mass larger than 2 GeV. Similar analysis was performed by DELPHI for the  $b$  quark-couplings to a scalar  $h$  and pseudoscalar  $A$ . New measurements of Yukawa process at LEP by OPAL and DELPHI groups [57], improve slightly these results, see discussion in Sec. IV.

A measurements of  $Z \rightarrow h(A)\gamma$  performed by all experimental groups at LEP I was used to obtain upper limits (however weaker than from the Yukawa processes) and lower limits on the Yukawa couplings  $\chi_d$  [56]. Still large part of the parameter space remains unconstrained.

Also the dedicated global fit to the EW precision data in the 2HDM (II) framework allows for an existence of very light scalar or pseudoscalar [59], for a partly constrained the heavy Higgs boson sector, including the  $H$  and  $H^\pm$  bosons. Note that the lower mass limit of  $H^\pm$  estimated from the direct search at LEP is 78.6 GeV [50]. The  $b \rightarrow s\gamma$  data interpreted in the 2HDM (II) give, according to the newest results [68],  $M_{H^\pm} > 320$  GeV or even higher [69]. One of the important message from the global fit based on the EW precision data [59] is that for a light  $h$  and large  $\tan\beta$ , and for mass of  $M_H$  below 1 TeV, an upper limit for  $M_{H^\pm}$  can be derived. In order to agree with the above lower mass limit based on  $b \rightarrow s\gamma$  data,  $\tan\beta$  should be smaller than 22 (28) for a light  $h$  with mass 10(20) GeV. For the upper mass of  $H$  equal to 5 TeV, these maximal value of  $\tan\beta$  increases by  $\sim 3$ .

In light of the above results one can conclude that there is still a possibility of the existence of one light neutral Higgs particle  $h$  or  $A$  with mass even below  $\sim 40$ –50 GeV. Since for a very light  $h$  the limit  $\sin^2(\beta - \alpha) \ll 1$  should hold, the second scalar,  $H$ , is expected to mimic the SM Higgs boson couplings, as discussed in Sec. III A. Therefore it is reasonable to expect that its mass is equal

to  $\sim 115$  GeV (or slightly higher).

Other low energy measurements do not contradict such scenarios. The  $\eta$  decay data [71] exclude only a very light  $h$ , with mass below 280 MeV. The Wilczek process,  $\Upsilon \rightarrow h(A)\gamma$  [72] [104], has been measured by few groups [74, 75, 76, 77]. Unfortunately the corresponding predictions have large theoretical uncertainties both due to the QCD and relativistic corrections, see [48, 73]. In addition, in some experimental analyses the production of the Higgs boson was treated according to the general 2HDM while for a decay of such Higgs boson the SM rates were assumed, *e.g.* [76]. All of these measurements of the  $\Upsilon$  decays give only upper limits for a coupling of the Higgs boson to  $b$  quark. Although for mass below 10 GeV these limits seem to be stronger than others mentioned above, large uncertainties of various sources make difficult to use these limits on a similar footing as the LEP ones. In the present analysis on  $g-2$  data, in which as we will see below also *lower limits* for the Yukawa coupling appear, the data for the Wilczek process, even with large uncertainties, will play an important role in closing low mass (below 10 GeV) part of a parameter space for 2HDM (II). In this analysis we apply three different constraints from the  $\Upsilon \rightarrow h(A)\gamma$  process, from [75] (denoted in figures as K), [76] (N) and [77] (L).

Unfortunately there are no limits from the HERA collider on very light Higgs bosons [60, 61]. There are important upper limits for the Yukawa couplings  $\chi_d$  for  $h$  and  $A$  from the TEVATRON data in the large mass region. They were obtained originally for the Higgs bosons in MSSM [70]: for mass say 70 (120) GeV  $\tan\beta$  should be above 34 (60) [70]. These limits, rescaled by a factor  $\sqrt{2}$ , should hold also for  $h$  and  $A$  in context of the 2HDM (II).

In Fig. 1 we present the existing 95% CL limits for a Yukawa coupling  $\chi_d$  for  $h$  (solid line) and  $A$  (dashed line) in the 2HDM (II). Upper limits are from the Yukawa process (data from ALEPH [54] for  $A$  and DELPHI [55] for  $h$  and  $A$ ). Lower limits from the  $Z \rightarrow h(A)\gamma$  processes [56] measured at LEP are also shown. In addition the upper 90% CL constraints from the  $\Upsilon$  decay from [75] (K) are shown, rescaled by a factor 2 to take into account the preference of the Higgs boson to decay into tau leptons (It occurs in 2HDM (II) for the considered mass range, for  $\chi_d$  bigger than 2.). The TEVATRON limits [70] (with a factor  $\sqrt{2}$ ) are displayed as well.

The one-loop analysis based on the previous  $g-2$  for the muon results [66] led to a slight improvement of the upper limits of  $\chi_d$  as compared to the Yukawa processes, for  $M_{h(A)} \leq 2$  GeV (not shown). The new data improve constraints of the 2HDM(II) considerably, what will be shown below.

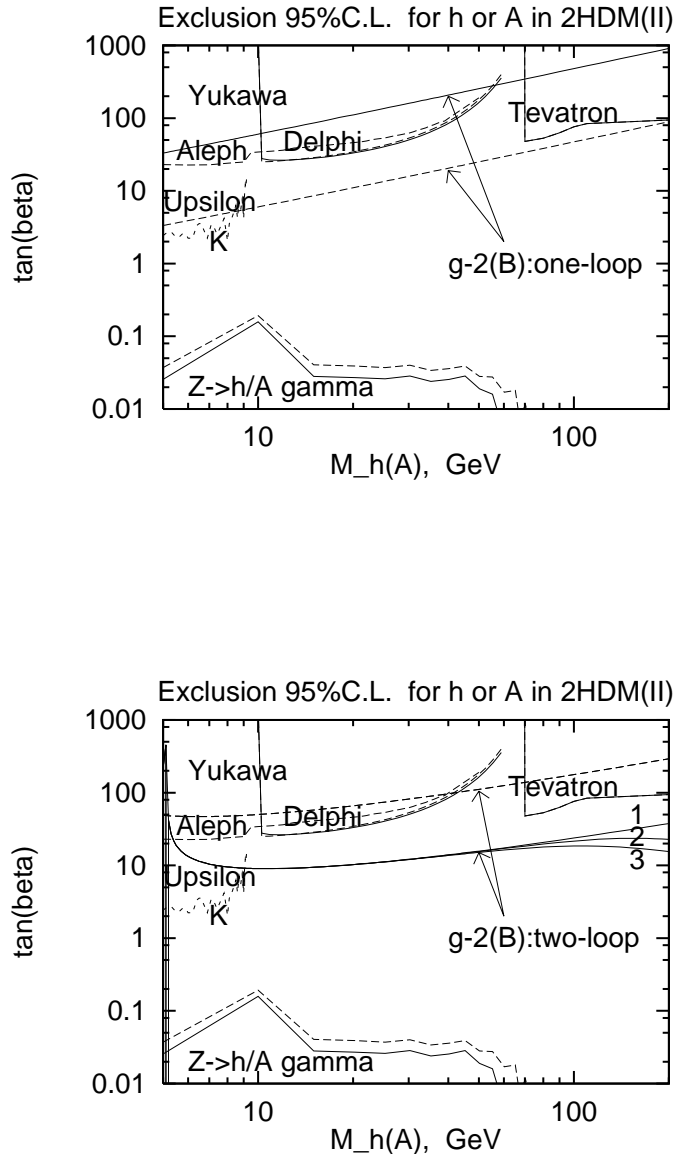


FIG. 1: The present upper and lower limits (95% CL) for the Yukawa coupling  $\chi_d$ , for a scalar  $h$  ( $\tan\beta$  if  $\chi_V^h = 0$ , solid line) and pseudoscalar  $A$  ( $\tan\beta$ , dashed line) as a function of mass. Upper limits are from the Yukawa process (ALEPH and DELPHI results) and lower limits from the  $Z \rightarrow h(A)\gamma$  processes measured at LEP. In addition the upper 90% CL constraints from the  $\Upsilon$  decay (K), rescaled by a factor 2 are shown. TEVATRON upper limits, both for  $h$  and  $A$ , are presented. The lines denoted “ $g-2(B)$ ” corresponds to the upper limits for  $h$  or  $A$  obtained in this paper for case B and  $\lim(95\%)$ : in the upper panel one-loop results, in the lower one the results of the two-loop analysis (for  $\chi_V^h = 0$ ), see text for details.

### C. 2HDM(II) contribution to $a_\mu$

In the 2HDM (II) neutral scalars  $h$  and  $H$ , pseudoscalar  $A$ , as well the charged Higgs boson  $H^\pm$  can contribute to  $a_\mu$ . There exist relevant calculations at the one-loop level [12, 13, 14] and the two-loop level [17, 80, 81, 87], see also earlier papers [15, 16]. In contrast to the one-loop approach where each Higgs boson exchange is given by a separate diagram (Fig. 2), various Higgs particles may appear in the same two-loop diagram, see Fig. 5(right).

We assume that the lightest Higgs boson,  $h$  or  $A$ , dominates the full 2HDM (II) contribution, *i.e.*  $a_\mu^{2HDM} \approx a_\mu^h$ , or  $a_\mu^A$  (*a simple approach*, see also [66]). This approach should hold for masses below 50 GeV, as discussed in Sec. III.B. For higher masses, which also are considered here, this should be treated as an assumption of a large gap between Higgs bosons masses. Since the charged Higgs boson mass should be bigger than 320 GeV [69], we do not consider  $H^\pm$  to be a lightest particle of the model.

We calculate separately the one-loop and two-loop contributions to both  $a_\mu^h$  and  $a_\mu^A$ . This way the importance of the two-loop diagrams can be seen, moreover our results can be easily compared with other one-loop calculation, *e.g.* [78]. For a pseudoscalar  $A$  the two-loop contributions are due to diagrams with fermion loops only. For a scalar the fermionic,  $W$  and a charged Higgs boson loops can contribute. However, the  $W$  and charged Higgs boson contributions are expected to be strongly suppressed for a small mass of  $h$ , where, according to the LEP data, the coupling  $\chi_V^h$  should be small (see discussion in Sec. III.B). For a simplicity in calculating of the two-loop contribution for  $h$  we explicitly assume for a whole mass range  $\chi_V^h = \sin(\beta - \alpha) = 0$ , what means a domination of the fermionic loops in  $a_\mu^h$ . The more extensive consideration will be given elsewhere [96].

For one- and two-loop approach we derive constraints on Yukawa coupling for  $h$  and  $A$  obtained from the estimated  $\delta a_\mu$  intervals (Sec. II.B), by taking  $a_\mu^{2HDM} = \delta a_\mu$ .

#### 1. One-loop calculation

The set of the relevant diagrams is presented in Fig. 2.

*a. Individual contributions.* The relevant one-loop formulae from the Appendix of paper [66], based on results [12, 13, 14], are given by ( $\Lambda = h, A$  or  $H^\pm$ )

$$a_\mu^\Lambda|_{\text{one-loop}} = \frac{f_\Lambda^2}{8\pi^2} L_\Lambda, \quad f_\Lambda \equiv \frac{g m_\mu}{2 M_W} \chi_d^\Lambda.$$

If  $\beta = \alpha$ , the coupling  $\chi_d^\Lambda$  is universal for  $h, A$  and  $H^\pm$ , and it is equal to  $\tan\beta$  (see sec. III. A).

The integral  $L_{h(A)}$  for the neutral Higgs boson contribution is given by (with  $z = (m_\mu/M_\Lambda)^2$ ):

$$L_{h(A)}(z) = z \int_0^1 dx \frac{Q_{h(A)}(x)}{x^2 z + (1-x)}.$$

with:  $Q_h(x) = x^2(2-x)$ ,  $Q_A(x) = -x^3$ . The charged Higgs particle exchange is described by:

$$\mathcal{L}_\pm(z) = z \int_0^1 dx \frac{-x(1-x)}{(x-1)z+1}.$$

The scalar contribution  $a_\mu^h$  [105] is positive whereas the pseudoscalar and the charged Higgs boson give negative contributions. Each one-loop contribution  $a_\mu^\Lambda$  disappears in the limit of large mass like  $m_\mu^2/M_\Lambda^2 \ln(M_\Lambda^2/m_\mu^2)$ , see also [65, 78]. At small mass each contribution reaches its maximum (or minimum if negative) value.

The individual (absolute value of) contributions, with couplings as in the SM, *i.e.* with  $\chi_d^\Lambda = 1$ , are shown in Fig.3. For mass above 0.2 GeV the charged Higgs contribution is much smaller than the contributions due to neutral Higgs bosons  $h$  and  $A$ . One observes that the  $h$  and  $A$  give practically the same contribution for mass larger than few GeV, only difference being in sign. In principle one should take into account a possible cancellation of these contributions, especially for masses above 50 GeV, see [96].

*b. Constraints.* First we present constraints based on the standard 95% CL intervals,  $\lim(95\%)$ . For case B this approach leads to the *upper limits* on the Yukawa coupling  $\chi_d$  for a pseudoscalar ( $\tan\beta$ ) and for a scalar ( $\tan\beta$  if  $\beta = \alpha$ ). Results are shown in Fig. 1 (upper) as lines denoted “ $g-2(B)$ :one-loop”. For case A one obtains an *allowed band* for the Yukawa coupling of a scalar only (only  $h$  gives a positive contribution). It can be found in Fig. 4 (only lower edge is shown) and in Fig.6 (upper), as a region between lines denoted A/B and A. A pseudoscalar is excluded in this case.

Both for B and A case, the limits for  $\chi_d$  rise with mass of the Higgs particle, what reflects the decrease of the corresponding  $a_\mu^{h,A}$  terms with increasing mass. For case A, only large  $\tan\beta$  (for  $\beta = \alpha$ ) greater than 10 is allowed for mass of  $h$  above 5 GeV, see Fig. 4.

Next we discuss results based on  $\lim_+(95\%)$  interval. For case A one obtains the *allowed band* for the Yukawa coupling  $\chi_d^h$ , which practically overlap with the considered above  $\lim(95\%)$  band, presented in Figs.4 and 6(upper). Now, also for case B instead of the upper limits an *allowed band* for the Yukawa coupling appears for  $h$

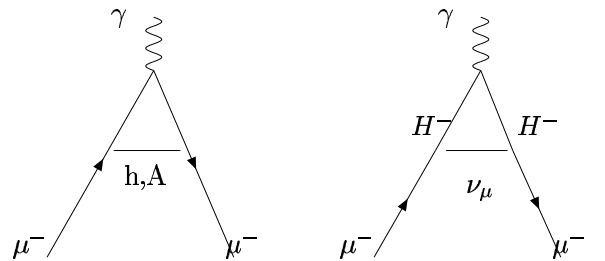


FIG. 2: One-loop contribution to  $g-2$  for muon due to a neutral scalar  $h$  (or  $H$ ), pseudoscalar  $A$  and a charged Higgs boson  $H^\pm$  exchange.

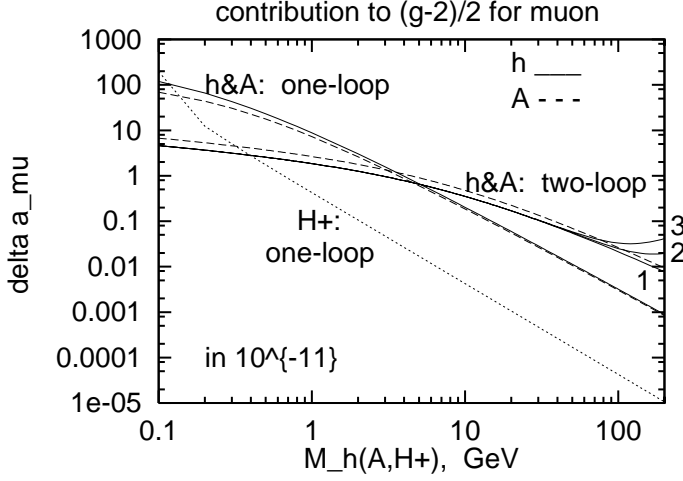


FIG. 3: The (absolute value of) individual contributions to  $a_\mu$  from a neutral scalar  $h$  (solid line), a pseudoscalar  $A$  (dashed line) and a charged Higgs boson  $H^+$  (dotted line). For results based on one-loop calculation:  $A$  and  $H^+$  contributions are negative. Two-loop diagram contributions only for  $A$  and  $h$  (denoted “1”) are based on the down-type fermion loops. For  $h$  also results with additional charged Higgs boson loop are shown (coupling of  $H^+$  equal to a first term in eq.6. with  $\mu = 0$ ) line “2”(“3”) corresponds to  $M_{H^+}=800(400)$  GeV. The two-loop  $h$  ( $A$ ) contribution is negative (positive). Yukawa couplings as in the SM are assumed.

(only). In Fig. 6 (upper) these two allowed bands for  $h$  are compared: a region between lines A/B and A, and a wider region between lines A/B and  $B_+$ . For both cases, A and B, one can exclude the negative contribution, as they may be realized at the level 0.5 % or 2.8%, respectively. This means an *exclusion of a pseudoscalar*.

## 2. Two-loop calculation

The two-loop diagrams, see Fig. 5, can give large contributions since they allow to avoid one small Yukawa coupling with muon in favor of the coupling with other, potentially heavy, particles circulating in the loop [15, 16, 80, 81, 87]. In addition to the mass effect, such contribution can be enhanced further by an additional factors (eqs. 1-3,5).

In principle diagrams with a  $Z$  boson, instead of the exchanged photon, may appear. However they are expected to be small [80] and are neglected in this analysis.

Below we consider the diagrams with fermionic loops Fig. 5 (left), which are the only two-loop contributions for a pseudoscalar  $A$ . For  $h$  in addition also a charged

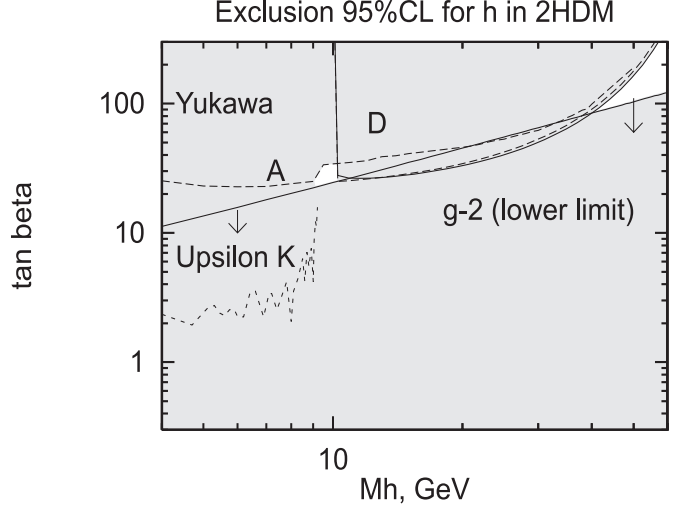


FIG. 4: The exclusion plot for a Yukawa coupling  $\chi_d^h$  ( $\tan \beta$  for  $\beta = \alpha$ ) for a scalar in the 2HDM (II) (a one-loop calculation). The lower limit from the  $(g-2)_\mu$  data for case A, the allowed region lays above the line with arrows. Upper limits from the Yukawa process (A=ALEPH and D=DELPHI), and the  $\Upsilon$  decay (K). White areas are allowed at 95% CL (both  $lim$  and  $lim_+$ ).

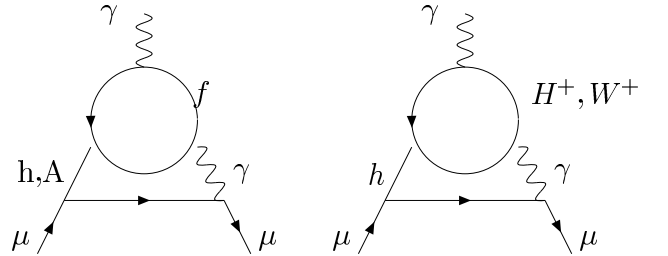


FIG. 5: Two-loop contributions to  $a_\mu$  from a light  $h$  or  $A$  with a fermionic loop (left); two-loop diagram for a light  $h$  with a charged Higgs boson  $H^+$  or  $W^+$  loop (right).

Higgs boson loop (Fig. 5 (right)) is taken into account. In calculation of the two-loop contributions we take  $\chi_V^h=0$ , as discussed above. With this condition one can neglect a  $W$ -loop contribution.

*a. Individual contributions.* The contributions from diagrams with fermionic loops presented in Fig.5 (left) are given by the following formulae for  $\Lambda = h, A$  [80, 81, 87] [106]:

$$a_\mu^\Lambda|_{two-loop} = \frac{f_\Lambda^2}{8\pi^2} \frac{e^2}{\pi} \xi_\kappa \tilde{L}_\Lambda^f, \quad f_\Lambda \equiv \frac{g}{2} \frac{m_\mu}{M_W} \chi_d^\Lambda. \quad (7)$$

The  $\kappa$  parameter is equal to 1 for a pseudoscalar  $A$ , for a scalar  $h$  we have

$$\kappa = \frac{\chi_u^h}{\chi_d^h} \text{ for } f = \text{"}u\text{-type quarks"}, \quad (8)$$

$$\kappa = 1 \text{ for } f = \text{"}d\text{-type quarks"}$$

$$\text{for } f = \text{"}charged\ leptons\text{"}.$$

The  $\xi$  parameter is equal to 1 for leptons and to  $N_c Q_q^2$  for a quark  $q$  with the electric charge  $Q_q e$ ,  $N_c = 3$ .

The integral  $\tilde{L}_{h(A)}^f$  for the neutral Higgs boson contribution with a fermionic loop is given by:

$$\tilde{L}_{h(A)}^f(z) = \frac{z}{2} \int_0^1 dx \frac{\tilde{Q}(x)_{h(A)}}{x(1-x)-z} \ln \frac{x(1-x)}{z}, \quad (9)$$

$$z = \left( \frac{m_f}{M_{h/A}} \right)^2,$$

and  $\tilde{Q}_h(x) = -(1 - 2x(1-x))$ ,  $\tilde{Q}_A(x) = 1$ . A diagram with the  $H^+$  loop presented in Fig.5 (right) contributes to  $a_\mu^h$  with the integral  $\tilde{L}_h^{H^+}(x)$  given by a similar expression as for fermionic loop (eq.11), with  $\tilde{Q}_h^{H^+}(x) = -(x(1-x))$  [87]. The corresponding coefficients for its contribution to  $a_\mu^h$  are:  $\xi = 1$  and  $\kappa = \chi_{H^+}^h / \chi_d^h$ .

The integrals describing two-loop contribution with a fermionic loop is negative for  $h$ , while positive for  $A$ , on contrary to the one-loop results. In Fig. 3 the (absolute value of) two-loop fermionic contributions, with  $\mu, \tau, b$  loops for  $h$  (denoted "1", solid line) and  $A$  (dashed line), are presented for  $\chi_d = 1$ . These two-loop contributions dominate for masses above few GeV over the corresponding (absolute value of) one-loop terms for both  $h$  and  $A$ . They have a milder than the one-loop contribution dependence on the mass of  $h$  (or  $A$ ), namely  $\ln(m_f^2/M_h^2)$ . From the figure one can read that the change of the sign of the sum of the one- and two-loop terms appears at mass of  $A$  ( $h$ ) equal to 3 (5) GeV.

The  $top$  contribution dominates for  $\chi_u = 1$  in the large mass region, being of order  $\sim 1 \cdot 10^{-11}$  (not shown). However it does not play an important role in the present analysis, since its contribution is proportional to  $\chi_d \chi_u$  equal to -1 for the pseudoscalar  $A$  and (if  $\chi_V^h=0$ ) also for  $h$ . So, the  $top$ -loop contribution has no additional enhancement factor, and for large  $\chi_d$  the  $d$ -type fermion loops dominate, both in  $a_\mu^h$  and  $a_\mu^A$ .

The charged Higgs boson loop contribution gives, for  $\chi_V^h=0$  and  $\kappa = \frac{M_h^2 - \mu^2}{2M_{H^+}^2} > 0$ , a negative contribution to  $a_\mu^h$ . This contribution (absolute value of) rises with a mass of  $h$  like  $M_h^2/M_{H^+}^2 (\ln M_{H^+}^2/M_h^2 + 5/3)$ . Its effect can be seen at  $M_h$  above 100 GeV. In Fig.3 these results for  $a_\mu^h$  are presented for  $\mu = 0$  (lines "2" for  $M_{H^+} = 800$  GeV and "3" (400 GeV)). It is clear that a charged Higgs boson loop with the parameters as described above leads to a small modification at large mass of  $h$  only. More detailed discussion will be given elsewhere [96].

*b. Constraints.* A full two-loop calculation, with both one- and two-loop diagrams included, leads to results which differ significantly from the ones based on the one-loop diagrams only. The main difference is related to the fact that for masses above 5 (3) GeV, a scalar (pseudoscalar) contribution to  $a_\mu$  has opposite sign as compared to the corresponding one-loop contribution. It means that now for each scenario, with a light  $h$  and a light  $A$ , the contribution can be positive or negative depending on mass. It is not the case for the one-loop terms where the corresponding contributions have a fixed sign.

In the derivation of the constraints in two-loop approach we take also into account the non-leading terms, namely the  $top$  contribution and for  $h$  also a term  $\sim \chi_u \chi_d$  due to the charged Higgs boson loop (6). The results are as follows.

The case B allows in the standard approach for a negative and positive contributions to  $a_\mu$ . Therefore the upper limits for  $\chi_d$  exist for the whole mass ranges for both  $h$  and  $A$ . In Fig. 1 (lower) we present the obtained limits (lines denoted "g-2(B):two-loop") for a mass region above 5 GeV. Here the upper limit for pseudoscalar was obtained from the maximal positive  $\delta a_\mu$  value, while for a scalar – from a maximal negative one, see the first table in Sec. II.B. This explains why a scalar is now constrained more tightly than a pseudoscalar, contrary to limits based on the one-loop calculation (Fig. 1 (upper)). For a scalar results obtained with only down-type fermions  $\mu, \tau, b$  included in the two-loop calculation are represented by the line denoted "1". Lines "2" and "3" are obtained if in addition one takes into account the charged Higgs boson-loop, as described above. Note that if one compares the two-loop constraints for a scalar to the one-loop constraints for a pseudoscalar these for a scalar are more tight for mass above few GeV, due to a weaker mass dependence of the  $a_\mu^h|_{two-loop}$ , see Fig. 3.

The case A leads to an allowed band for a positive contribution to  $a_\mu$ , *i.e.* for a scalar  $h$  with mass below 5 GeV, and for a pseudoscalar  $A$  with mass above 3 GeV, similar results were found in [80, 81]. Obtained constraints are presented in Fig.6 (lower) and in Fig. 7 as the regions between lines A/B and A. In the same figures also results for case B obtained with  $\lim_+(95\%)$  are shown (regions between lines A/B and  $B_+$ ). It is not clear however whether this approach should be used here, as for both, a light  $A$  and a light  $h$ , scenarios, both positive and negative contributions to  $a_\mu$  are possible, as discussed in Sec. III. C.

## D. Combined 95% CL constraints of the 2HDM (II)

When the above constraints obtained from a new  $g-2$  for the muon measurement are added to the existing constraints from other processes discussed in Sec. III. B. (and also in [57]) interesting conclusions can be reached in the 2HDM (II) for both scenarios: with a scalar  $h$  and

a pseudoscalar  $A$  being the lightest particles in the model. We start discussion of the results based on one-loop calculation which can be compared with similar analysis [78], then results of the two-loop analysis are presented.

The constraints are obtained for (absolute value of)  $\chi_d$ , which is equivalent to  $\tan\beta$  for pseudoscalar and for scalar, provided in the latter case  $\chi_V^h$  is equal or close to zero. Only in the calculation of the two-loop contribution we explicitly use the assumption  $\chi_V^h = 0$ . For a simplicity of the discussion we will use below the  $\tan\beta$  to represent the Yukawa coupling  $\chi_d^h$  for a scalar in all cases.

### 1. Allowed regions from one-loop results.

The upper limits for  $\tan\beta$  for a pseudoscalar  $A$  which were obtained from  $g-2$  data for case B are much tighter than the limits from other experiments for mass above 10 GeV. Still a window with a light  $A$  is open for  $\tan\beta$  below  $\sim 10$  (Fig. 1(upper)). For a scalar  $h$  a weak improvement is observed only for a mass range between 60-70 GeV where  $\tan\beta$  has to be lower than  $\sim 300$ .

The  $(g-2)_\mu$  results for case A rule out a pseudoscalar, for a scalar they improve considerably existing limits. The obtained an allowed band in the  $(\tan\beta, M_h)$  plane, is equivalent to existence of both the upper (as above) and also the lower limits for the Yukawa coupling. An allowed by all experiments region appears for a scalar  $h$  with mass between 40 and 70 GeV at  $\tan\beta$  above 80 (Fig. 6 (upper)). In addition a small region of mass of  $h$  around 10 GeV and  $\tan\beta$  between 25 and 35 remains allowed (Fig. 4), see also [78] where the same result was obtained. Note, however, that this small allowed area can be closed by taking into account the mentioned in Sec. III.B constraint from the global fit [59]. This constraint, for the charged Higgs boson mass above 300 GeV, forbids  $\tan\beta$  to be larger than 22 for  $M_h=10$  GeV [107] [108].

At the one-loop level it is sensible to study  $\lim_+(95\%)$  intervals, since two scenarios of the 2HDM(II), with a light  $h$  or  $A$ , give contributions of a definite sign independently on mass. Pseudoscalar, giving a negative contribution, is ruled out. An allowed  $\lim_+(95\%)$  band for case B obtained for a scalar is wider than for discussed above case A (compare the region between lines A/B and  $B_+$  with the region between A/B and A in Fig.6 (upper)). Therefore also the allowed region in the  $(\tan\beta, M_h)$  plane is much larger for case B than for case A. It starts at mass 10 GeV and stops at 70 GeV with  $\tan\beta$  between 10 and  $\sim 300$ .

In Fig. 6 (upper) we plot also an the expected upper 95 % CL limits from process  $gg \rightarrow h \rightarrow \tau\tau$  at the  $ep$  collider HERA (dotted lines for lower and higher luminosity), from [60, 61]. These measurements may help to cover the low mass region for  $h$  and  $A$  at the intermediate Yukawa coupling  $\tan\beta$ .

### 2. Allowed regions from two-loop results.

The two-loop diagrams give dominant contributions to  $a_\mu^{2HDM}$  for mass above 3 GeV (5 GeV) for a pseudoscalar (scalar) and these contributions have reversed signs relative to the one-loop results. Two-loop analysis for these regions (based on a sum of the one- and two-loop diagram contributions) leads to similar conclusions as the one-loop one, with a reversed role of a scalar and a pseudoscalar. In particular now the (standard 95% CL) upper limits obtained for case B for scalar, for  $M_h > 10$  GeV, are much tighter in comparison with limits from other experiments (Fig. 1 (lower)). Still a window with a light  $h$  is open for  $\tan\beta$  below 10. For a pseudoscalar the improvement is weak, being limited to a mass region between 40-70 GeV with  $\tan\beta \leq 100$ .

The constraints obtained in case A in a form of allowed bands improve considerably the existing up to now limits for both scalar and pseudoscalar since in addition to the mentioned above upper limits also the lower limits appear (Fig. 6 (lower) and Fig.7). The allowed by  $(g-2)_\mu$  data bands for a scalar  $h$  is situated now below  $M_h=5$  GeV. This region is however excluded by the  $\Upsilon$  decay data (Fig. 6 (lower)). On the other hand a pseudoscalar with mass above 10 GeV and  $\tan\beta$  larger than 20 is still in agreement with existing data (see Fig.7). The TEVATRON data close practically the region of the mass above 70 GeV for a case A.

If for case B the  $\lim_+$  interval is applied this region of mass  $M_A$  above 70 GeV is still open.

### 3. Discussion

As  $(g-1)_\mu$  data favor a positive additional contribution it is clear that at one-loop level the most stringent limits of the new  $g-2$  data for the muon can be driven for a pseudoscalar, the case A leads even to its exclusion.

The two-loop calculation leads to a radical change of the picture. The two-loop diagrams give the dominant contributions for mass above 3 GeV (5 GeV) for a pseudoscalar (scalar) and with a reversed sign as compared to the one-loop results. Constraints are now tighter for a scalar.

We point out a role in the present analysis of the low energy measurement of the Wilczek process  $\Upsilon \rightarrow h/A\gamma$  in closing the window for mass below 10 GeV for both a scalar and pseudoscalar. The TEVATRON results close a part of the large mass and large  $\tan\beta$  region. The global fit adds an important constraints for  $h$  with mass around 10 GeV. New measurements of Yukawa process at LEP by OPAL and DELPHI [57], which were presented recently do not change our qualitatively our conclusions, see also below.

#### IV. CONCLUSION

We described the room for a new effects as follows from the recent  $(g - 2)_\mu$  measurements and from new theoretical estimations of  $a_\mu^{SM}$ . For two SM predictions arising from different values of  $a_\mu^{had}$ , case A (DH) and case B (J2000), we evaluate 95% CL intervals for a new contribution. They can be used to constrain parameters of any model beyond the Standard Model. We show how these constraints depend on the size of the  $a_\mu^{had}$  (case A and B), and on the type of limits ( $lim$  or  $lim_+$ ). The *upper limits* for the negative and positive contributions, obtained if the prediction B and the  $lim(95\%)$  are used in the analysis, are to be contrasted with the *allowed band*, obtained in case A for a positive contribution only. For both A and B cases a negative  $\delta a_\mu$  contribution is very unlikely: a positive (negative) contribution corresponds to 99.5 (0.5) % CL for A, while for case B to 97.2 (2.8) % CL. At this level models leading to only negative  $\delta a_\mu$  can be excluded, and vice versa models which lead to the positive contribution only should be accepted as possible models. For such models the derived  $lim_+(95\%)$  estimations differ from those based on  $lim(95\%)$  considerably for case B.

We applied the obtained intervals to constrain parameters of the 2HDM (II) using a simple approach, where only one Higgs boson,  $h$  or  $A$ , contributes. In the one-loop calculation a light scalar scenario leads to the positive, whereas the one with a light pseudoscalar to the negative contribution to  $a_\mu$ , independently of mass. In the two-loop analysis, based on a sum of the one- and two-loop diagram contributions, the situation changes drastically. Now the positive contribution can be ascribed to a scalar  $h$  with mass below 5 GeV or a pseudoscalar  $A$  with mass above 3 GeV. In both, one- and two-loop,

approaches we derive tight constraints on the Yukawa couplings to muon of  $h$  and  $A$ . When these constraints are combined with constraints arising from other experiments, especially the Wilczek process and the Yukawa processes at LEP and TEVATRON, large part of the parameter space for light  $h$  or light  $A$  can be excluded.

Our results obtained for the case B lead to an improved upper limits for both  $h$  and  $A$ . For a more constraining case A our results including all other existing constraints are as follows (in parenthesis the limits obtained if the newest DELPHI data [57] are included):

The one-loop calculation excludes a pseudoscalar while allows for an existence of a light scalar  $h$  with mass above 40 GeV (50 GeV) and below 70 GeV, and  $\tan\beta$  larger than 90.

The two-loop analysis allows for an existence of a pseudoscalar with mass between  $\sim 10$  GeV (25 GeV) and 70 GeV, and  $\tan\beta$  above 20 (30). A light scalar being excluded by combining the  $(g - 2)_\mu$  and the  $\Upsilon$  decay data.

#### V. ACKNOWLEDGMENTS.

I am grateful to Jan Żochowski for his collaboration at the early stage of this work, and to Piotr Zalewski for his important suggestions. I thank Marek Szczekowski, Piotr Chankowski and Stefan Pokorski for helpful discussions on this analysis and Andrzej Czarnecki and William Marciano for clarification of the newest results for  $g - 2$  for muon. I am grateful to Bohdan Grzadkowski and Wai-Tee Keung for a discussion on two-loop calculation and to Fred Jegerlehner for useful information. I am indebted also to T. and P.-M. Krawczyk for their help in preparation of this paper. Supported in part by the Polish Committee for Scientific Research, Grants 5 P03B 121 20 i 2 P03B 05119, and the European Commission 50th framework contract HPRN-CT-2000-00149.

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- [1] H. N. Brown *et al.* [Muon  $g-2$  Collaboration], Phys. Rev. Lett. **86**, 2227 (2001) [hep-ex/0102017].
  - [2] A. Czarnecki and W. J. Marciano, Phys. Rev. D **64**, 013014 (2001) [hep-ph/0102122].
  - [3] F. J. Yndurain, hep-ph/0102312.
  - [4] W. J. Marciano and B. L. Roberts, hep-ph/0105056.
  - [5] S. Narison, Phys. Lett. B **513**, 53 (2001) [hep-ph/0103199].
  - [6] F. Jegerlehner, hep-ph/0104304.
  - [7] K. Melnikov, hep-ph/0105267.
  - [8] J. F. De Troconiz and F. J. Yndurain, hep-ph/0106025.
  - [9] J. Prades, hep-ph/0108192.
  - [10] A. Czarnecki and W. J. Marciano, Nucl. Phys. B (Proc. Suppl.) **76**, 245 (1999); P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**, 351 (2000);
  - [11] A. I. Studenikin, Phys. Atom. Nucl. **62** (1999) 2071 [Yad. Fiz. **62** (1999) 2248], hep-ph/9808219.
  - [12] R. Jackiw and S. Weinberg, Phys. Rev. D **5**, 2396 (1972). K. Fujikawa *et al.*, Phys. Rev. D **6**, 2923 (1972). G. Altarelli *et al.*, Phys. Lett. B **40**, 415 (1972). W. A. Bardeen *et al.*, Nucl. Phys. B **46**, 315 (1972).
  - [13] J. Leveille, Nucl. Phys. B **137**, 63 (1978).
  - [14] H. E. Haber *et al.*, Nucl. Phys. B **161**, 493 (1979).
  - [15] J. D. Bjorken and S. Weinberg, Phys. Rev. Lett. **38**, 622 (1977).
  - [16] S. M. Barr and A. Zee, Phys. Rev. Lett. **65**, 21 (1990) [Erratum-ibid. **65**, 2920 (1990)].
  - [17] J. A. Grifols and R. Pascual, Phys. Rev. D **21**, 2672 (1980).
  - [18] T. Kinoshita *et al.*, Phys. Rev. D **41**, 593 (1990); T. Kinoshita, Phys. Rev. D **47**, 5013 (1993); M. Samuel and G. Li, Phys. Rev. D **44**, 3935 (1991) (Erratum D **48**, 1879 (1993)); S. Laporta and E. Remiddi, Phys. Lett. B **301**, 440 (1992); S. Laporta, Phys. Lett. B **312**, 495 (1993); S. Karshenboim, Yad. Fiz. **56**, 252 (1993); S. Laporta and E. Remiddi, PL **B** 379, 283 (1996) (hep-ph/9602417).
  - [19] T. Kinoshita, Phys. Rev. Lett. **75**, 4728 (1995). V. W. Hughes and T. Kinoshita, Rev. Mod. Phys. **71** (1999) S133.
  - [20] J. Calmet, S. Narison, M. Perrottet and E. de Rafael, Rev. Mod. Phys. **49**, 21 (1977). J. Calmet, S. Narison,

- M. Perrottet and E. de Rafael, *Phys. Lett. B* **61**, 283 (1976). S. Narison, *J. Phys. G* **G4**, 1849 (1978).
- [21] L. M. Barkov *et al.*, *Nucl. Phys. B* **256**, 365 (1985).
- [22] T. Kinoshita, B. Nizic and Y. Okamoto, *Phys. Rev. D* **31**, 2108 (1985).
- [23] T. Kinoshita and W.J. Marciano, in *Quantum Electrodynamics*, ed. by T. Kinoshita (World Scientific, Singapore, 1990), pp. 419-478; T. Kinoshita, *Z. Phys. C* **56**, S80 (1992).
- [24] L. Martinovic and S. Dubnicka, *Phys. Rev. D* **42**, 884 (1990).
- [25] E. de Rafael, *Phys. Lett. B* **322**, 239 (1994) [hep-ph/9311316].
- [26] S. Eidelman and F. Jegerlehner, *Z. Phys. C* **67**, 585 (1995) [hep-ph/9502298]. F. Jegerlehner, in *Proc. of the Workshop "QCD and QED in Higher Orders"*, Rheinsberg, Germany, 1996 (hep-ph/9606484).
- [27] F. Jegerlehner, 2000, seminar at the New York University in honor of A. Sirlin's 70th Birthday.
- [28] R. Alemany, M. Davier and A. Hocker, *Eur. Phys. J. C* **2**, 123 (1998) [hep-ph/9703220].
- [29] M. Davier and A. Hocker, *Phys. Lett. B* **435**, 427 (1998) [hep-ph/9805470].
- [30] M. Davier, hep-ph/9912044; *Nucl. Phys. (Proc. Supp.) B* **76**, 327 (1999).
- [31] K. Adel and F. J. Yndurain, hep-ph/9509378. J. A. Casas, C. Lopez and F. J. Yndurain, *Phys. Rev. D* **32**, 736 (1985).
- [32] D. H. Brown and W. A. Worstell, *Phys. Rev. D* **54**, 3237 (1996) [hep-ph/9607319].
- [33] J. Erler and M. Luo, *Phys. Rev. Lett.* **87**, 071804 (2001) [hep-ph/0101010].
- [34] V. Cirigliano, G. Ecker and H. Neufeld, *Phys. Lett. B* **513**, 361 (2001) [hep-ph/0104267].
- [35] B. Krause, *Phys. Lett. B* **390**, 392 (1997) [hep-ph/9607259].
- [36] M. Hayakawa *et al.*, *Phys. Rev. Lett.* **75**, 790 (1995); *Phys. Rev. D* **54**, 3137 (1996), (hep-ph/9601310).
- [37] J. Bijnens, E. Pallante and J. Prades, *Nucl. Phys. B* **474**, 379 (1996) [hep-ph/9511388].
- [38] M. Hayakawa and T. Kinoshita, *Phys. Rev. D* **57**, 465 (1998) [hep-ph/9708227].
- [39] E. Bartos, A. Z. Dubnickova, S. Dubnicka, E. A. Kuraev and E. Zemlyanaya, hep-ph/0106084.
- [40] S.J. Brodsky and J.D. Sullivan, *Phys. Rev.* **156**, 1644 (1967), T. Burnet and M. J. Levine, *Phys. Lett.* **24B**, 467 (1967) I. Bars and M. Yoshimura, *Phys. Rev. D* **6**, 374 (1972).
- [41] W. J. Marciano and A. Sirlin, *Phys. Rev. Lett.* **61**, 1815 (1988).
- [42] W. J. Marciano, *Phys. Rev. D* **45**, 721 (1992).
- [43] T. V. Kukhto, E. A. Kuraev, Z. K. Silagadze and A. Schiller, *Nucl. Phys. B* **371**, 567 (1992).
- [44] A. Czarnecki, B. Krause and W. J. Marciano, *Phys. Rev. D* **52**, 2619 (1995) [hep-ph/9506256].
- [45] A. Czarnecki, B. Krause and W. J. Marciano, *Phys. Rev. Lett.* **76**, 3267 (1996) [hep-ph/9512369].
- [46] S. Peris, M. Perrottet and E. de Rafael, *Phys. Lett. B* **355**, 523 (1995) [hep-ph/9505405].
- [47] G. Degrossi and G. F. Giudice, *Phys. Rev. D* **58**, 053007 (1998) [hep-ph/9803384].
- [48] J. F. Gunion, H. E. Haber, G. Kane and S. Dawson, "The Higgs Hunter's guide" Addison-Wesley (1990);
- [49] I. F. Ginzburg, M. Krawczyk and P. Osland, hep-ph/0101208.
- [50] G. Hanson, talk at Lepton-Photon 2001
- [51] G. Abbiendi *et al.* [OPAL Collaboration], *Eur. Phys. J. C* **18**, 425 (2001) [hep-ex/0007040].
- [52] P. Abreu *et al.* [DELPHI Collaboration], *Eur. Phys. J. C* **17**, 187 (2000).
- [53] The L3 Collaboration, M. Acciarri *et al.*, *Phys. Letters B* **385** (1996) 454.
- [54] The ALEPH Collaboration, Search for a Light Higgs Boson in the Yukawa Process, PA13-027, Contribution to the International Conference on High Energy Physics, Warsaw, Poland, 25-31 July 1996.
- [55] The DELPHI Collaboration, Search for Yukawa production of a light neutral Higgs at LEP1, DELPHI 99-76 CONF 263, Paper submitted to the HEP'99 Conference, Tampere, Finland, July 15-21.
- [56] M. Krawczyk, J. Zochowski and P. Mattig, *Eur. Phys. J. C* **8**, 495 (1999) [hep-ph/9811256].
- [57] DELPHI Coll., results presented at LP2001 and EPS Conferences; OPAL Coll., results presented at LP2001 and EPS Conferences
- [58] OPAL Coll., S. Soldner-Rembold, CERN, 10 July 2001
- [59] P. H. Chankowski, M. Krawczyk and J. Zochowski, *Eur. Phys. J. C* **11**, 661 (1999) [hep-ph/9905436].
- [60] A. C. Bawa and M. Krawczyk, *Phys. Lett. B* **357**, 637 (1995). M. Krawczyk and B.B. Levchenko, *Two-photon mechanism production of the Higgs boson, SUSY particles, hadrons and lepton pairs in eA collision at HERA*, in proc. Workshop "Future of HERA" 1995-1996 p. 978;
- [61] M. Krawczyk, *Higgs search at HERA*, in proc. Workshop "Future of HERA" 1995-1996 p. 244, hep-ph/9609477.
- [62] D. Choudhury and M. Krawczyk, *Phys. Rev. D* **55**, 2774 (1997) [hep-ph/9607271].
- [63] M. Krawczyk, P. Mattig and J. Zochowski, *Eur. Phys. J. C* **19**, 463 (2001) [hep-ph/0009201].
- [64] S. Kanemura, T. Kasai and Y. Okada, *Phys. Lett. B* **471**, 182 (1999) [hep-ph/9903289].
- [65] E. D. Carlson, S. L. Glashow and U. Sarid, *Nucl. Phys. B* **309** (1988) 597.
- [66] M. Krawczyk and J. Zochowski, *Phys. Rev. D* **55**, 6968 (1997) [hep-ph/9608321].
- [67] M. Krawczyk, *Acta Phys. Polon. B* **29**, 3543 (1998) [hep-ph/9812493].
- [68] The CLEO Collaboration, S. Alam, *Phys. Rev. Letters* **74** (1995) 2885; The CLEO Collaboration, CONF 98-17, ICHEP98 1011, submitted to the ICHEP conference Vancouver, 1998; The ALEPH Collaboration, R. Barate *et al.*, *Physics Letters B* **429** (1998) 169; M. Misiak, S. Pokorski and J. Rosiek, *Heavy Flavours II*, eds. A.J. Buras and M. Lindner, p. 795 (hep-ph/9703442); M. Ciuchini *et al.*, *Nucl. Phys. B* **527** (1998), 21. F. Blanc, EW Moriond 2001; S. Chen *et al.* [CLEO Collaboration], hep-ex/0108032.
- [69] M. Misiak, private communication; P. Gambino and M. Misiak, hep-ph/0104034.
- [70] M. Roco [CDF and D0 Collaborations], FERMILAB-CONF-00-203-E.
- [71] J. Prades and A. Pich, *Phys. Letters B* **245**, 117 (1990)
- [72] F. Wilczek, *Phys. Rev. Lett.* **39**, 1304 (1977).
- [73] M. I. Vysotsky, *Phys. Lett. B* **97**, 159 (1980). P. Na-

- son, Phys. Lett. B **175**, 223 (1986). S. N. Biswas, A. Goyal and J. Pasupathy, Phys. Rev. D **32**, 1844 (1985). G. Faldt, P. Osland and T. T. Wu, Phys. Rev. D **38**, 164 (1988). I. G. Aznaurian, S. G. Grigorian and S. G. Matinian, JETP Lett. **43**, 646 (1986). Yad. Fiz. **45**, 152 (1987). Phys. Lett. B **214**, 637 (1988).
- [74] Crystal Ball Coll., D. Antresyan et. al., *Phys. Lett. B* **251**, 204 (1990); CLEO Coll., R. Balest et al., CLEO 94-19; CUSB Coll., P. Franzini et al., *Phys. Rev. D* **35**, 2883 (1987).
- [75] S. M. Keh, *Tau physics with the Crystal Ball Detector*, DESY F31-86-6.
- [76] M. Narain, PhD Thesis, *Inclusive Photon Spektra from  $\Upsilon$  Decays*, 1991
- [77] J. Lee-Franzini, talk at ICHEP 1988, Munich, in proc. p. 1432
- [78] A. Dedes and H. E. Haber, JHEP **0105**, 006 (2001) [hep-ph/0102297].
- [79] F. Larios, G. Tavares-Velasco and C. P. Yuan, Phys. Rev. D **64**, 055004 (2001) [hep-ph/0103292].
- [80] D. Chang, W. Chang, C. Chou and W. Keung, Phys. Rev. D **63**, 091301 (2001) [hep-ph/0009292].
- [81] K. Cheung, C. Chou and O. C. Kong, hep-ph/0103183.
- [82] Y. Wu and Y. Zhou, hep-ph/0104056.
- [83] J. L. Lopez, D. V. Nanopoulos and X. Wang, Phys. Rev. D **49**, 366 (1994) [hep-ph/9308336]. U. Chattopadhyay and P. Nath, *Phys. Rev. D* **53**, 1648 (1996).
- [84] T. Moroi, Phys. Rev. D **53**, 6565 (1996) [hep-ph/9512396]. M. Carena, G. F. Giudice and C. E. Wagner, Phys. Lett. B **390**, 234 (1997) [hep-ph/9610233]. K. T. Mahanthappa and S. Oh, hep-ph/9909410. K. T. Mahanthappa and S. Oh, Phys. Rev. D **62**, 015012 (2000) [hep-ph/9908531]. G. Cho, K. Hagiwara and M. Hayakawa, Phys. Lett. B **478**, 231 (2000) [hep-ph/0001229].
- [85] L. L. Everett, G. L. Kane, S. Rigolin and L. Wang, Phys. Rev. Lett. **86**, 3484 (2001) [hep-ph/0102145]. J. L. Feng and K. T. Matchev, Phys. Rev. Lett. **86**, 3480 (2001) [hep-ph/0102146]. T. Ibrahim, U. Chattopadhyay and P. Nath, Phys. Rev. D **64**, 016010 (2001) [hep-ph/0102324]. J. R. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B **508**, 65 (2001) [hep-ph/0102331]. K. Choi, K. Hwang, S. K. Kang, K. Y. Lee and W. Y. Song, Phys. Rev. D **64**, 055001 (2001) [hep-ph/0103048]. S. P. Martin and J. D. Wells, Phys. Rev. D **64**, 035003 (2001) [hep-ph/0103067]. H. Baer, C. Balazs, J. Ferrandis and X. Tata, Phys. Rev. D **64**, 035004 (2001) [hep-ph/0103280]. G. Cho and K. Hagiwara, Phys. Lett. B **514**, 123 (2001) [hep-ph/0105037]. W. de Boer, M. Huber, C. Sander and D. I. Kazakov, hep-ph/0106311. G. Cho, hep-ph/0107169. M. Byrne, C. Kolda and J. E. Lennon, hep-ph/0108122. T. Blazek, R. Dermisek and S. Raby, hep-ph/0107097. Y. Daikoku, hep-ph/0107305. A. Dedes, H. K. Dreiner and U. Nierste, hep-ph/0108037. E. Gabrielli, K. Huitu and S. Roy, hep-ph/0108246.
- [86] C. Chen and C. Q. Geng, Phys. Lett. B **511**, 77 (2001) [hep-ph/0104151].
- [87] A. Arhrib and S. Baek, hep-ph/0104225.
- [88] S. Baek, T. Goto, Y. Okada and K. Okumura, hep-ph/0104146. Z. Chacko and G. D. Kribs, hep-ph/0104317.
- [89] E. A. Baltz and P. Gondolo, Phys. Rev. Lett. **86**, 5004 (2001) [hep-ph/0102147]. U. Chattopadhyay and P. Nath, Phys. Rev. Lett. **86**, 5854 (2001) [hep-ph/0102157]. R. Arnowitt, B. Dutta, B. Hu and Y. Santoso, Phys. Lett. B **505**, 177 (2001) [hep-ph/0102344]. Y. G. Kim and M. M. Nojiri, hep-ph/0104258. G. Belanger, F. Boudjema, A. Cottrant, R. M. Godbole and A. Semenov, hep-ph/0106275.
- [90] D. A. Dicus, H. He and J. N. Ng, Phys. Rev. Lett. **87**, 111803 (2001) [hep-ph/0103126]. E. Mituda and K. Sasaki, hep-ph/0103202. Z. Xing, Phys. Rev. D **64**, 017304 (2001) [hep-ph/0102304]. J. Hisano and K. Tobe, Phys. Lett. B **510**, 197 (2001) [hep-ph/0102315]. R. Adhikari, E. Ma and G. Rajasekaran, hep-ph/0108167. T. Blazek and S. F. King, hep-ph/0105005.
- [91] K. Lane, hep-ph/0102131. D. Chakraverty, D. Choudhury and A. Datta, Phys. Lett. B **506**, 103 (2001) [hep-ph/0102180]. K. R. Lynch, hep-ph/0108080, hep-ph/0108081. D. Choudhury, B. Mukhopadhyaya and S. Rakshit, Phys. Lett. B **507**, 219 (2001) [hep-ph/0102199]. S. N. Gninenko and N. V. Krasnikov, Phys. Lett. B **513**, 119 (2001) [hep-ph/0102222]. C. A. de S. Pires and P. S. Rodrigues da Silva, hep-ph/0103083, S. Baek, N. G. Deshpande, X. G. He and P. Ko, Phys. Rev. D **64**, 055006 (2001) [hep-ph/0104141]. hep-ph/0108200. C. S. Kim, J. D. Kim and J. Song, Phys. Lett. B **511**, 251 (2001) [hep-ph/0103127]. S. C. Park and H. S. Song, Phys. Lett. B **506**, 99 (2001) [hep-ph/0103072]. X. Calmet, hep-ph/0108079. Z. Xiong and J. M. Yang, Phys. Lett. B **508**, 295 (2001) [hep-ph/0102259]. C. Yue, Q. Xu and G. Liu, J. Phys. G **G27**, 1807 (2001) [hep-ph/0103084].
- [92] R. A. Diaz, R. Martinez and J. A. Rodriguez, Phys. Rev. D **64**, 033004 (2001) [hep-ph/0103050]. S. K. Kang and K. Y. Lee, hep-ph/0103064. E. O. Ilhan hep-ph/0103105.
- [93] S. Barshay and G. Kreyerhoff, hep-ph/0106047. H. Chavez, L. Masperi and M. Orsaria, hep-ph/0108183.
- [94] M. Graesser and S. Thomas, hep-ph/0104254. T. Ibrahim and P. Nath, hep-ph/0105025. J. L. Feng, K. T. Matchev and Y. Shadmi, hep-ph/0107182. R. Arnowitt, B. Dutta, B. Hu and Y. Santoso, hep-ph/0108082.
- [95] M. B. Einhorn and J. Wudka, Phys. Rev. Lett. **87**, 071805 (2001) [hep-ph/0103034].
- [96] P. Chankowski, M. Krawczyk, P. Zalewski- in preparation
- [97] Particle Data Group, *Phys. Rev. D* **50**, 1173 (1994).
- [98] V. Barger et al. *Phys. Rev. D* **41**, 3421 (1990), Y. Grossman, *Nucl. Phys. B* **426**, 355 (1994)
- [99] In our analysis from 1996 [66] the corresponding differences between the case A and B had only a quantitative character.
- [100] We found it difficult to apply the  $lim_-$  (95%) method to the negative contribution, which is outside (or almost outside in case B) the 95% region around  $\Delta a_\mu$ .
- [101] for the M analysis we added errors in quadrature
- [102] even for  $\tan\beta = 1$  they may differ by sign from the SM value
- [103] The perturbativity arguments constrain the  $\tan\beta$  range between  $\sim 0.2$  and 200-300 [98].
- [104] also a decay  $J/\psi \rightarrow h(A)\gamma$  [74]
- [105] For a larger mass it also can be used to represent  $H$ . Note, that the SM Higgs boson contribution (with mass  $\sim 150$  GeV) is included in the (two-loop) prediction for  $a_\mu^{SM}$ .

- [106] The two-loop contribution for a charged Higgs boson can be found in [17].
- [107] see Fig.9a from [59]
- [108] The newest data on the Yukawa process [57] support this finding.

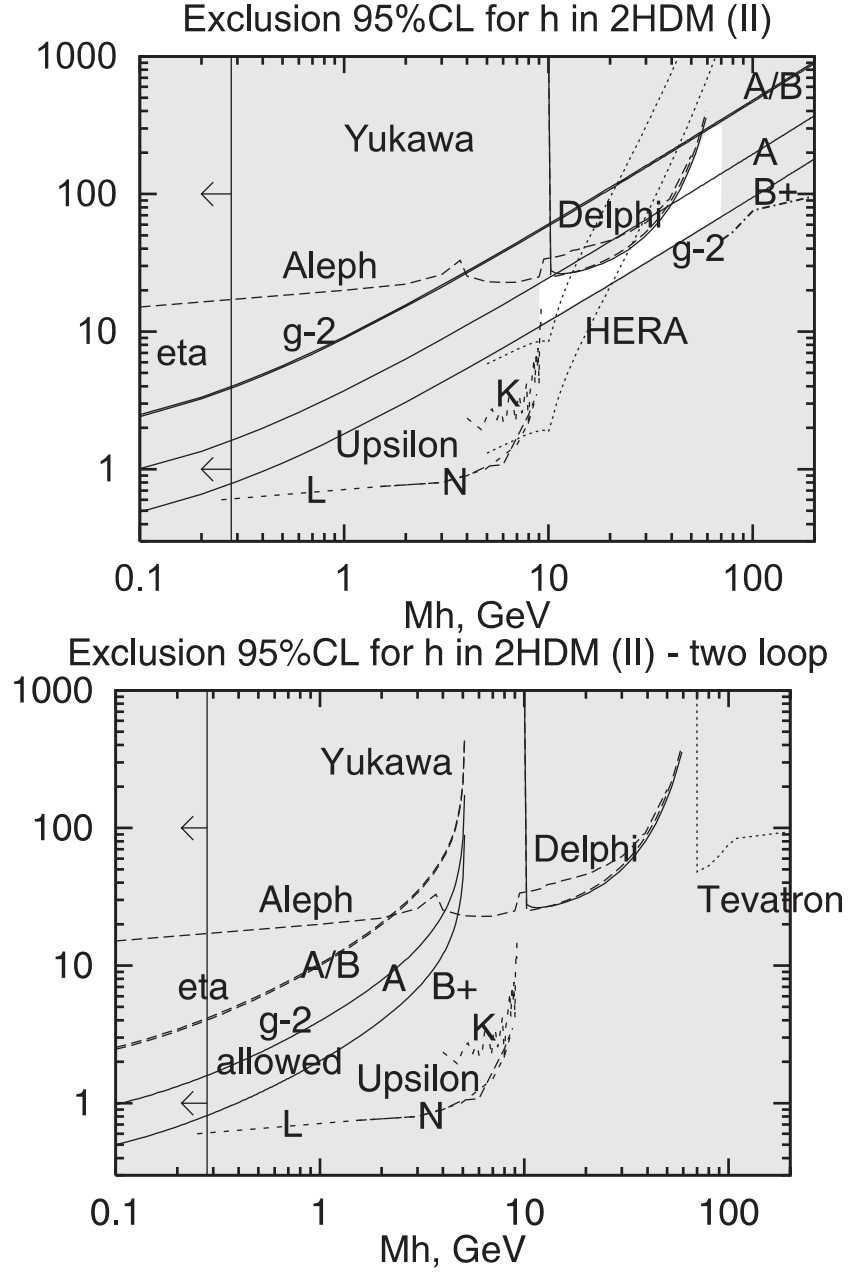


FIG. 6: Upper panel: one-loop result for  $h$ . Limits for the Yukawa coupling of the scalar  $h$  normalized to the SM value,  $\chi_d^h$  (equal to  $\tan\beta$  for  $\beta = \alpha$ ) as a function of the mass  $M_h$ . The constraint from the  $\eta$  decay excludes 95% CL the mass below 0.28 GeV (vertical line), upper 95% CL limits from the Yukawa process (ALEPH measurements for pseudoscalar - dashed line, DELPHI data for scalar (pseudoscalar) - solid (dashed) line). Upper 90% CL limits from the  $\Upsilon \rightarrow h/A\gamma$ , (K, N and L results, see text). Results from (K) are rescaled by a factor 2. The upper 95% limits from the TEVATRON collider for  $h$  and  $A$  (rescaled by  $\sqrt{2}$  for masses below 130 GeV) are also shown (dot-dashed line). Allowed  $lim(95\%)$  bands for  $h$  from the newest  $g - 2$  data for the muon: regions between the line corresponding to both A and B cases (denoted A/B) and lines giving lower bounds calculated for case A and case B ( $lim_+(95\%)$ ) (denoted B+), respectively. For a comparison the expected upper 95 % CL limits from process  $gg \rightarrow h \rightarrow \tau\tau$  at the  $ep$  collider HERA are shown (dotted lines for lower and higher luminosity). Lower panel: sum of one- and two-loop results for  $h$  (for  $\beta = \alpha$ ). Curves as in the upper panel.

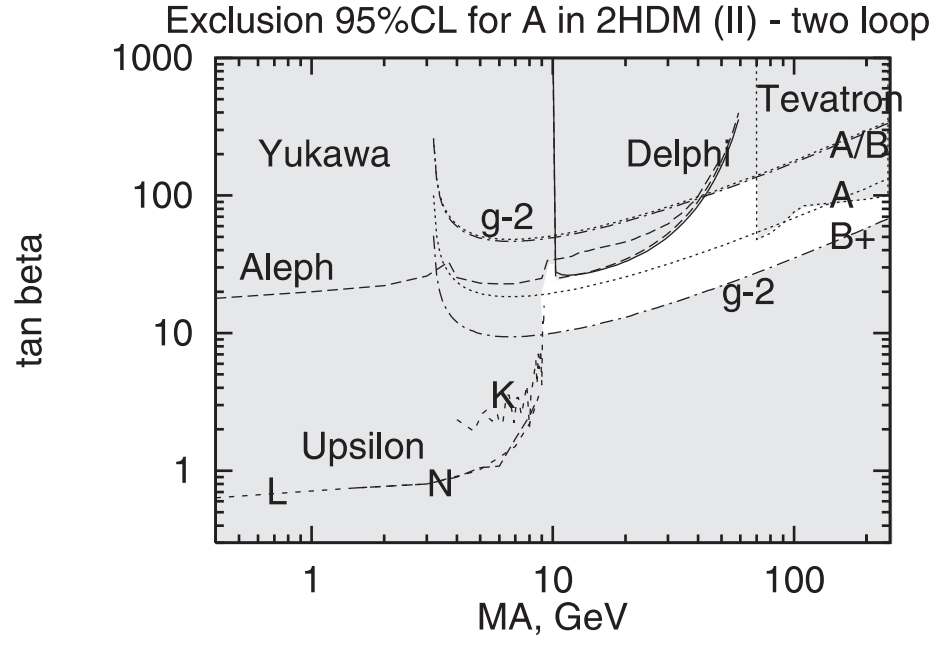


FIG. 7: Two-loop result for  $A$ . Limits for the Yukawa coupling of the pseudoscalar  $A$  normalized to the SM value,  $\chi_d^A$  equal to  $\tan \beta$ , as a function of the mass  $M_A$ . The allowed bands from  $g - 2$  data for  $A$  and  $B(\lim_+(95\%))$  cases are shown together with constraints from other experiments, details as in Figs.6. The white area is allowed by the all existing data.